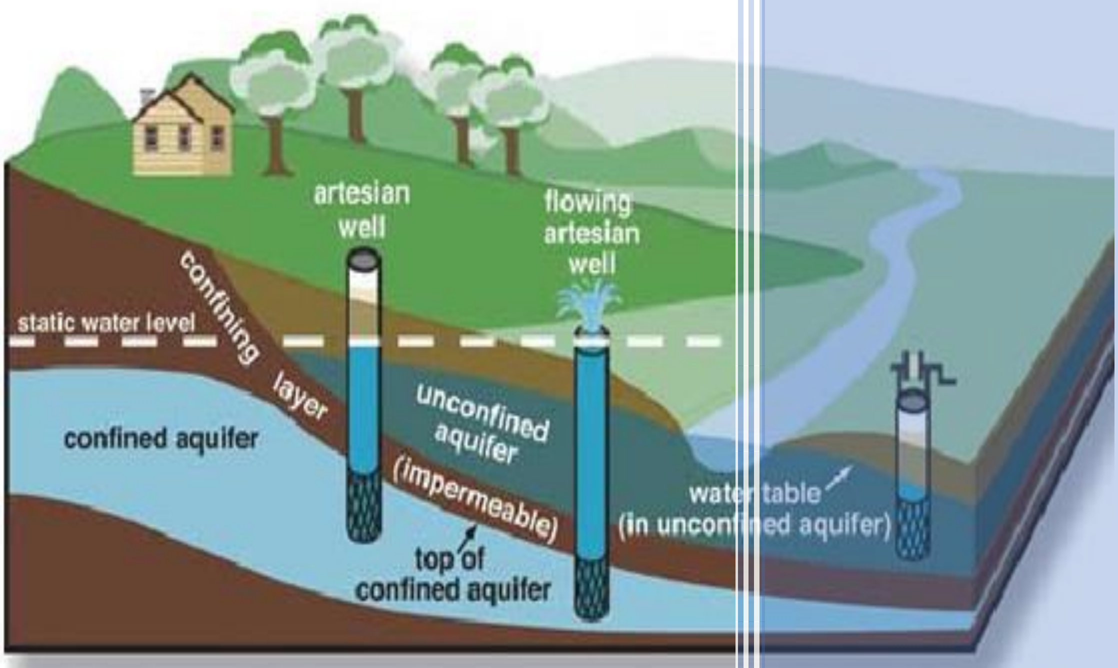
713, 2022

GROUNDWATER HYDROLOGY



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Lecture Notes, *according to* Chapters 14 and 15 *from the following reference:*

WATER-RESOURCES ENGINEERING

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Video Lectures

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CHAPTER 1

Fundamentals of Groundwater Hydrology I: Governing Equations

1. Introduction

Groundwater hydrology is the science dealing with the quantity, quality, movement, and distribution of water below the surface of the earth. The field of groundwater hydrology is sometimes called *geohydrology* or *hydrogeology*.

The major engineering applications of the principles of groundwater hydrology are:

- Developing of water supplies through wells and infiltration galleries.
- Evaluating, mitigating, and remediation of contaminated groundwater.
- Storing temporarily fresh water in underground reservoirs.
- Controlling groundwater levels to permit crop growth and facilitate subsurface construction.

Groundwater accounts for approximately 30% of all the fresh water on earth, as shown in Figure 1, which is second only to the polar ice (69%).

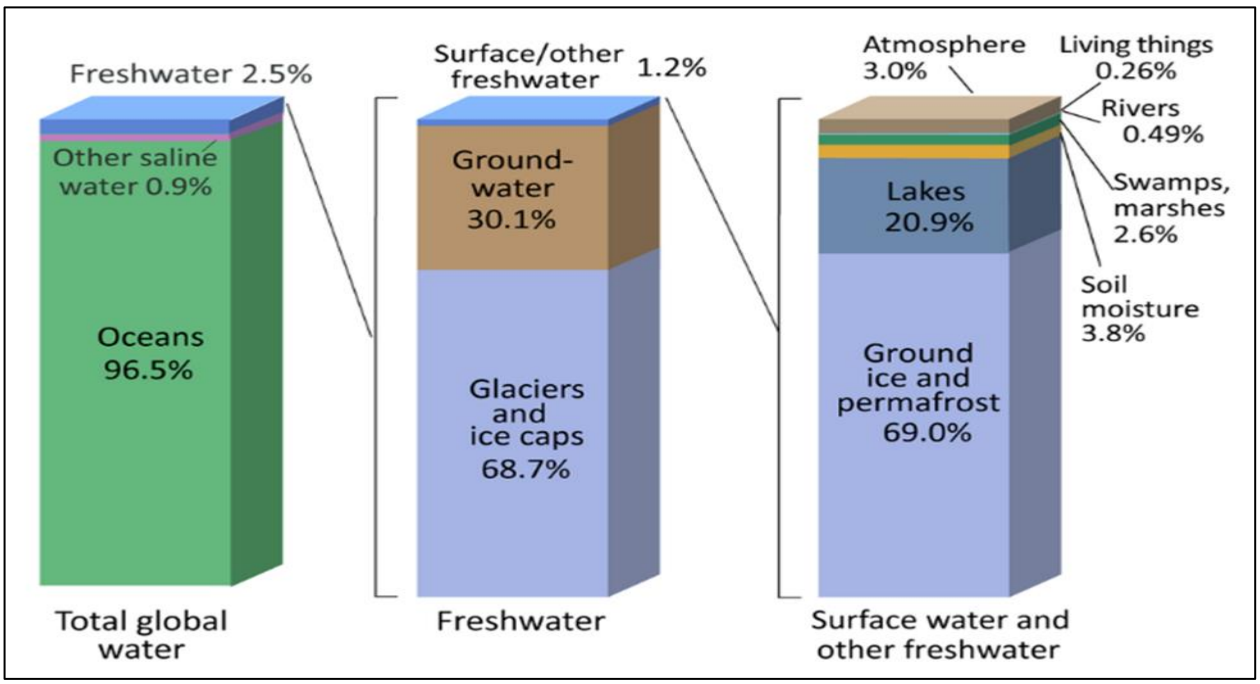


Figure 1. Ground Water Quantity

https://www.usgs.gov/special-topic/water-science-school/

The subsurface environment is a porous medium in which the void spaces have varying degrees of water saturation. The region where void spaces are filled with water is called the *saturation zone*, and the region where void spaces are not filled with water is called the *aeration zone*, as illustrated in Figure 2.

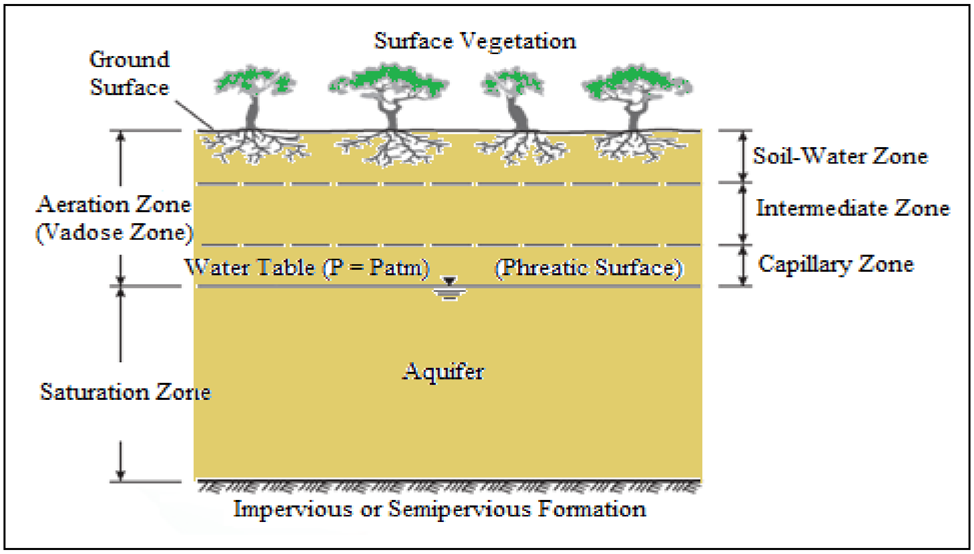


Figure 2. Ground Water Classification

Water in the aeration zone is sometimes called *vadose water*[1], and the aeration zone is sometimes called the *vadose zone*. Typically, the aeration zone (vadose zone) lies above the saturation zone, and the upper boundary of the saturation zone is called the *phreatic surface*[2] or *water table*. At the water table, the pressure is equal to atmospheric pressure.

Within the aeration zone are three subzones: the soil-water, intermediate, and capillary zones.

The *soil-water zone* is the region containing the roots of surface vegetation, voids left by decayed roots of earlier vegetation, and animal and worm burrows.

The maximum moisture content in the soil-water zone corresponds to the maximum moisture that can be held by the soil against the force of gravity, regardless of the depth of the water table below the ground surface. The maximum moisture content in the soil-water zone is called the *field capacity*, and the thickness of the soil-water zone is typically on the order of 1–3 m.

[1] *Vadose* is a derivative of the Latin word *vadosus*, which means “shallow”.
[2] *Phreatic* is a derivative of the Greek word *phreatos*, which means “well”. In this context, a saturated zone is encountered when a well is dug.

Beneath the soil-water zone is the *intermediate zone*, which extends from the bottom of the soil-water zone to the upper limit of the capillary zone.

The *capillary zone* extends from the water table up to the limit of the capillary rise of the water from the saturation zone. The thickness of the capillary zone depends on the pore sizes in the material above the water table and can vary from 1 cm for gravels to several meters for clays.

An *aquifer*[3] is a geologic formation containing water that can be withdrawn in significant amounts.

An aquifer consists of the entire water-bearing geologic unit (or the entire group of water-bearing units) and not just its saturated portion.

Aquicludes contain water but are incapable of transmitting it in significant quantities (e.g. clay layers), and can be taken as impervious formations.

Aquifuges neither contain nor transmit water (e.g. solid rocks).

Aquifers are classified as either unconfined or confined.

Unconfined aquifers are open to the atmosphere, as illustrated in Figure 3, and are also called *phreatic aquifers* or *water-table aquifers*.

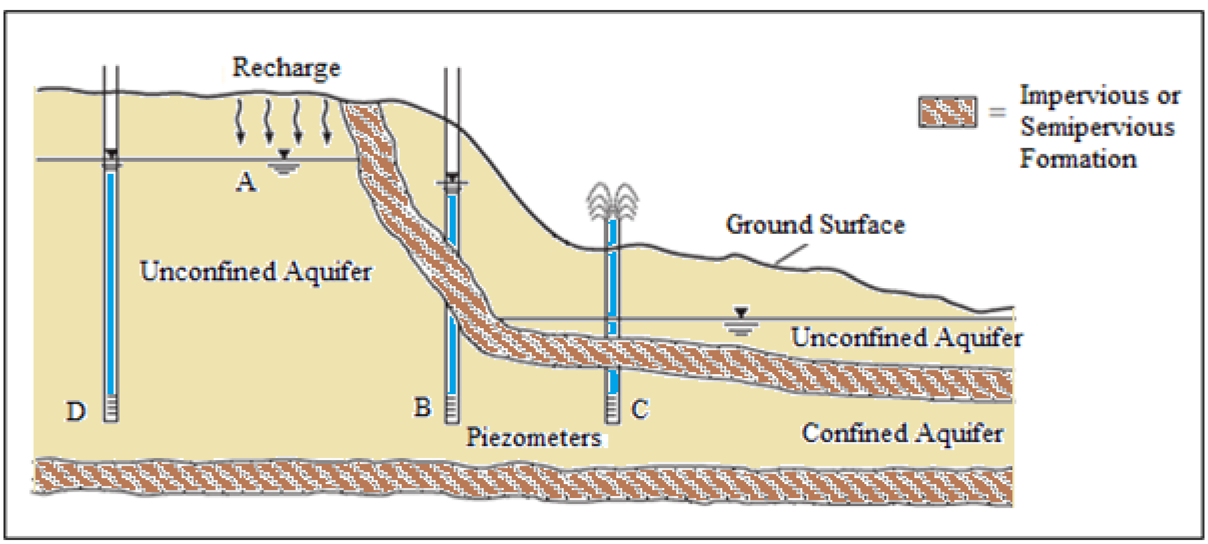


Figure 3. Confined, Unconfined, and Artesian Aquifers

[3] *Aquifer* is a derivative of the Latin words' *aqua* "water" and *ferre* "to bear".

In some cases, water accumulates above an elevated stratum of low permeability, and water above this stratum spills over into an unconfined aquifer. In such cases, the water table above the low-permeability stratum is called a *perched water table*, as illustrated in Figure 4.

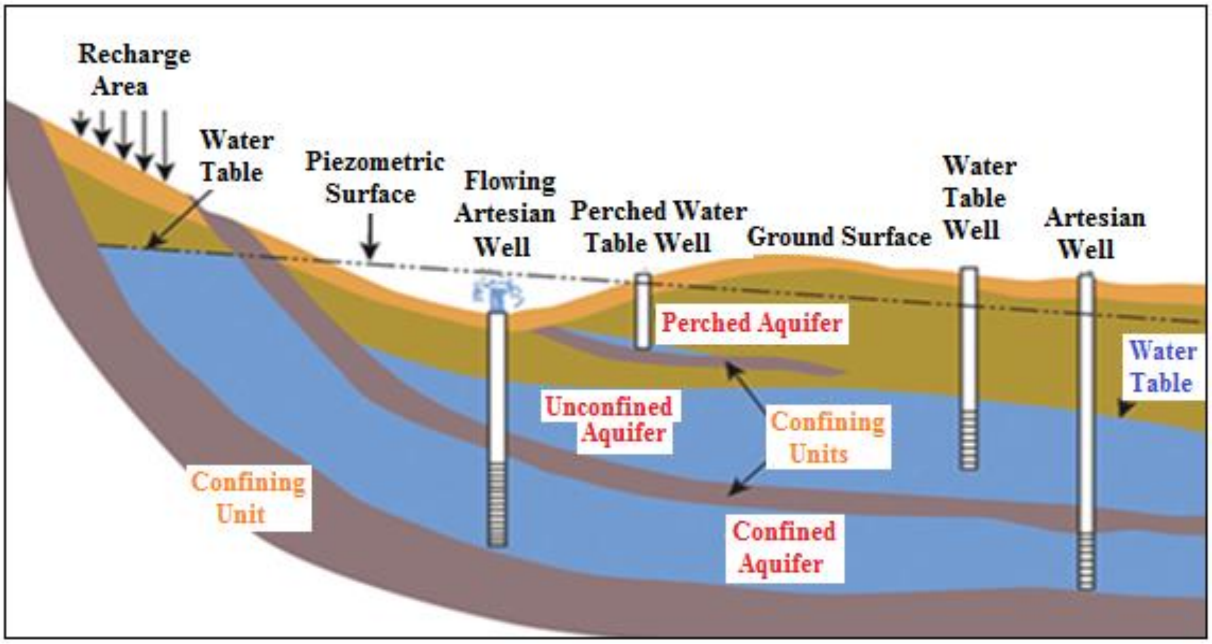


Figure 4. Aquifers and Wells
https://www.waternz.org.nz/Story?Action=View&Story_id=238 (*Modified*)

Clay lenses in sedimentary deposits often have shallow perched water bodies overlying them.

In *confined aquifers*, water in the saturated zone is bounded above by either impervious or semi-pervious formations, and water at the top of a confined aquifer is normally at a pressure greater than atmospheric pressure.

A typical configuration of a confined aquifer is shown also in Figure 3, where the water in the confined aquifer is recharged by inflows at A (usually from rainfall).

Land surfaces that supply water to aquifers are called *recharge areas*. Maintaining an adequate recharge area (and recharge water supply) is important in urban areas where groundwater is a major source of drinking water. The primary groundwater recharge mechanism is the infiltration of rainfall.

Piezometers are observation wells with very short-screened openings that are used to measure the piezometric head, φ, at the screened opening, which for an incompressible fluid is given by:

$$\varphi = \frac{p}{\gamma} + z \quad (1)$$

where p and z are the pressure and elevation, respectively, at the opening of the observation well (piezometer), and γ is the specific weight of the groundwater.

If the confined aquifer shown in Figure 3 is penetrated by piezometers at B and C, then the water levels in these piezometers rise to levels equal to the piezometric heads at B and C, respectively.
At B, the water in the piezometer rises above the top confining layer of the confined aquifer, indicating that the water pressure at the top of the confined aquifer is greater than atmospheric pressure.
For the case of a piezometer in an unconfined aquifer, such as at D, the water level in the piezometer rises to the water table.

The piezometer at C behaves similarly to the piezometer at B, the difference is that the water level in the piezometer at C rises above the ground surface. This means that if a well were extended from the ground surface down into the confined aquifer at C, then groundwater would flow continuously from the well until the piezometric head was reduced to the elevation at the top of the well.

An aquifer that produces flowing water when penetrated by a well from the ground surface is called an *artesian aquifer*[4]. As indicated in Figure 3, a confined aquifer can be an artesian aquifer at some locations, such as at C, and non-artesian at other locations, such as at B.

In cases where artesian aquifers intersect the ground surface, concentrated flows of groundwater called *springs* are formed.

Groundwater inflows into surface-water channels are a common source of perennial discharge in streams, which is referred to as the *base flow* of the stream. Stream flows mostly consist of the base flow plus the flow resulting from stormwater runoff.

Both unconfined and confined aquifers can be bounded by semi-pervious formations called *aquitards*, which are significantly less permeable than the aquifer but are not impervious.

Unconfined aquifers can only be bounded by impervious or semi-pervious layers on the bottom, while confined aquifers are bounded on both the top and bottom by either impervious or semi-pervious layers.

Aquifers bounded by semi-pervious formations are called *leaky aquifers*, and terms such as *leaky-unconfined aquifer* and *leaky-confined aquifer* are commonly used.

[4] The name “artesian” is derived from the name of the northern French city of Artois, where wells penetrating artesian aquifers are common.

A microscopic view of the flow through a porous medium is illustrated in Figure 5, where water flows through the *void space* and around the *solid matrix* within the porous medium.

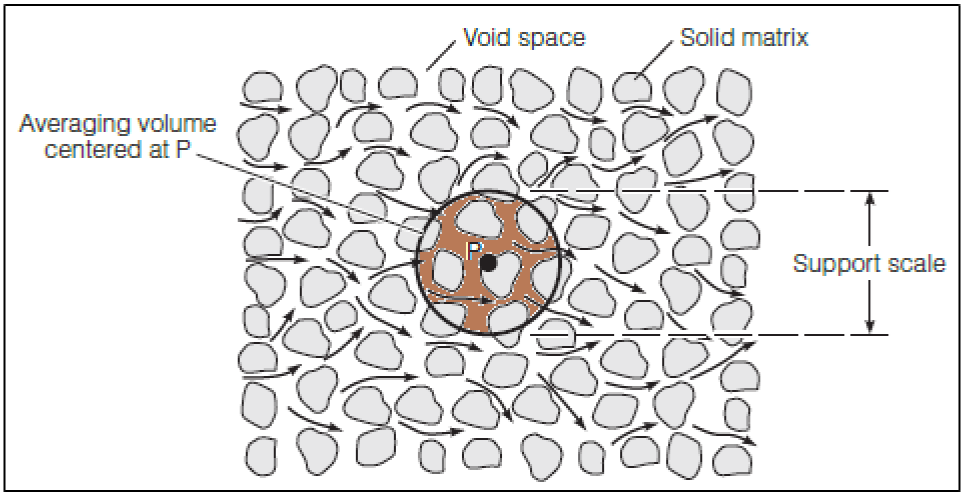


Figure 5. Microscopic View of Flow through a Porous Medium

It is difficult to describe the details of the flow field within the void spaces since this would require detailed knowledge of the geometry of the void space within the porous medium. To deal with this problem, it is convenient to work with spatially averaged variables rather than variables at a point.

Referring to Figure 5, a property of the porous medium at P can be taken as the average value of that property within a volume centered at P. The scale of the average volume is called the *support scale*. In general, the value of the average quantity is independent of the size of the support scale.

The *porosity*, *n*, of a porous medium <u>is defined by the relation:</u>

$$n = \frac{voids\ volume}{sample\ volume} \tag{2}$$

The sample volume can be taken as the spherical volume with a diameter equal to the support scale.

Representative values of porosity in consolidated formations are given in Table 1.

Table 1. Representative Hydrologic Properties in Consolidated Formations

Material	Porosity, n	Specific Yield	Hydraulic Conductivity, K, m/d
Sandstone	0.05 – 0.50	0.01 – 0.41	10^{-5} – 4
Limestone	0.00 – 0.56	0.00 – 0.36	10^{-4} – 2000
Schist (صخر متبلر)	0.01 – 0.50	0.20 – 0.35	10^{-4} – 0.2
Siltstone	0.20 – 0.48	0.01 – 0.35	10^{-6} – 0.001
Claystone	0.41 – 0.45	---	---
Shale (صخر طيني)	0.00 – 0.10	0.01 – 0.05	10^{-8} – 0.04
Till	0.22 – 0.45	0.01 – 0.34	10^{-5} – 30
Basalt	0.01 – 0.50	---	10^{-6} – 2000
Pumice (حجر الخفاف)	0.80 – 0.90	---	---
Tuff (حجر مسامي)	0.10 – 0.55	0.01 – 0.47	---

Between 60% and 90% of all developed aquifers consist of granular unconsolidated rocks[5], where the porosities are associated with the intergranular spaces determined by the particle-size distribution.

Granular material is classified by particle size distribution, and many different organizations have established classification standards for use in various disciplines.

The United States Department of Agriculture (USDA) soil classification system is one of the most widely used in water-resources engineering and is given in Table 2, along with corresponding values of porosity.

Porosity is considered small when $n < 0.05$, medium when $0.05 < n < 0.20$, and large when $n > 0.20$. [6]

The most common aquifer materials are unconsolidated sands and gravels, which occur in alluvial valleys, coastal plains, dunes, and glacial deposits.

Consolidated formations that make good aquifers are sandstones, limestones with solution channels, and heavily fractured volcanic and crystalline rocks.

Clays, shales, and dense crystalline rocks are the most common materials found in aquitards.

[5] Todd and Mays, 2005.
[6] Kashef, 1986.

Table 2. USDA Classification and Hydrologic Properties in Unconsolidated Formations

Classification*	Particle Size*, mm	Porosity, n	Specific Yield	Hydraulic Conductivity, K, m/d
Very coarse gravel	32 – 64	---	---	---
Coarse gravel	16 – 32	0.24 – 0.40	0.10 – 0.26	860 – 8600
Medium gravel	8 – 16	0.24 – 0.44	0.13 – 0.45	20 – 1000
Fine gravel	4 – 8	0.25 – 0.40	0.15 – 0.40	---
Very fine gravel	2 – 4	---	---	---
Very coarse sand	1.00 – 2.00	---	---	---
Coarse sand	0.50 – 1.00	0.20 – 0.50	0.15 – 0.45	0.08 – 860
Medium sand	0.25 – 0.50	0.29 – 0.49	0.15 – 0.46	0.08 – 50
Fine sand	0.10 – 0.25	0.25 – 0.55	0.01 – 0.46	0.01 – 40
Very fine sand	0.05 – 0.10	---	---	---
Silt	0.002 – 0.050	0.34 – 0.70	0.01 – 0.40	10^{-5} – 2
Clay	< 0.002	0.33 – 0.70	0.00 – 0.20	< 10^{-2}

Note: *USDA Soil Classification System for Sand, Silt, and Clay; while Gravel Classification is given by Morris and Johnson, 1967.

Aquifers range in thickness from less than 1 m to several hundred meters. They may be long and narrow, as in small alluvial valleys, or they may extend over millions of square kilometers.

The depth from the ground surface to the top of the saturated zone of an aquifer may range from 1 m to more than several hundred meters.

2. Darcy's Law

The study of flow through porous media was pioneered by Darcy[7] (1856) using an experimental setup like that shown in Figure 6. The motivation for Darcy's experiments was to study the performance of the sand filters in the water supply system for the city of Dijon, France.

In this experiment, Darcy investigated the flow rate of water through a column of sand with cross-sectional area A [L^2] and length L [L]. Darcy found that the volumetric flow rate, Q [L^3T^{-1}], of water through the sand column could be described by the relation:

$$Q = -K\,A\,\frac{\varphi_2 - \varphi_1}{L} \qquad (3)$$

[7] Henry Philibert Gaspard Darcy (1803–1858) was a French engineer.

where K is a proportionality constant [LT^{-1}], and φ_1 and φ_2 are the piezometric heads [L] at the entrance and exit of the sand column, respectively.

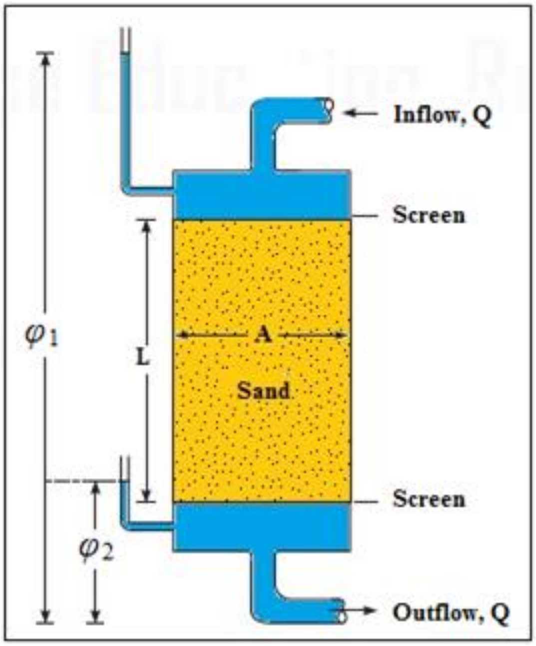


Figure 6. Schematic Diagram of Darcy's Experimental Setup

The piezometric head (*hydraulic head*), φ, of an incompressible fluid is given by $\varphi = p/\gamma + z$.

Defining the gradient in the piezometric head or *hydraulic gradient*, J [dimensionless], across the sand column by:

$$J = \frac{\varphi_2 - \varphi_1}{L} \tag{4}$$

The specific discharge, q [LT^{-1}], through the sand column by:

$$q = \frac{Q}{A} \tag{5}$$

Equation 3 can be written in the form:

$$q = -K\,J \tag{6}$$

which is commonly known as *Darcy's law*. The specific discharge, q, is sometimes called the *filtration velocity* or *Darcy's velocity*.

The experimentally validated relationship given by Equation 6 states that the specific discharge through a porous medium is linearly proportional to the gradient in the piezometric head in the direction of flow.

The proportionality constant, K, is called the *hydraulic conductivity* of the porous medium (*coefficient of permeability* in geotechnical engineering).

Before proceeding to study the applications of Darcy's law, several important points must be noted.

First, the specific discharge, q, is defined as the volumetric flow rate per unit cross-section of the porous medium and, since the flow only occurs within the pores, the actual velocity of flow through the pores is necessarily greater than the specific discharge.

The flow velocity through the pores is called the *seepage velocity*, v [LT^{-1}], and is defined by:

$$v = \frac{Q}{A_p} \qquad (7)$$

where Ap [L^2] is the area of the pores normal to the flow direction.

Comparing Equations 5 and 7, the seepage velocity, v, is related to the specific discharge, q, by:

$$v = q * \frac{A}{A_p} \qquad (8)$$

The ratio of the pore area, Ap, to the bulk cross-sectional area, A, is defined as the *areal porosity* and is equal to the volumetric porosity, n, which is defined by Equation 2.

However, not all the pore spaces are connected and available for fluid flow, therefore the effective areal porosity is less than the volumetric porosity, n. The ratio of the effective flow area to the bulk cross-sectional area is defined as the *effective porosity*, *ne*, as:

$$n_e = \frac{A_e}{A} \qquad (9)$$

where Ae is the effective flow area through the pores [L^2].

For unconsolidated porous media, the effective porosity is approximately equal to the volumetric porosity, while in many consolidated formations the effective porosity can be of magnitude smaller than the total porosity, with the greatest difference occurring in fractured rocks[8].

Combining Equations 8 and 9, and taking *Ap* equal to *Ae*, yields the following relationship between the seepage velocity, *v*, and the specific discharge, *q*, given by Darcy's law as:

$$v = \frac{q}{n_e} = -\frac{K}{n_e} J \qquad (10)$$

EXAMPLE 1

Water flows through a sand aquifer with a piezometric head gradient of 0.01. The hydraulic conductivity and effective porosity of the aquifer are 2 m/d and 0.3, respectively.

(a) Estimate the specific discharge and seepage velocity in the aquifer?
(b) Find the volumetric flow rate of the groundwater if the aquifer is 15 m deep and 1 km wide?
(c) How long does it take the groundwater to move 100 m?

Solution

(a) *From the given data:* $J = -0.01$, $K = 2$ m/d, $ne = 0.3$,
According to Darcy's law, the specific discharge, *q*, is:
$q = -K\,J = -(2)\,(-0.01) = 0.02$ m/d
The corresponding seepage velocity, *v*, is:
$v = q / ne = 0.02 / 0.3 = 0.067$ m/d

(b) The volumetric flow rate, *Q*, of groundwater across an area $A = 15$ m * 1 km = 15,000 m^2 is:
$Q = q\,A = (0.02)\,(15{,}000) = 300\ m^3/d$

(c) Groundwater flows with the seepage velocity, *v*, and therefore the time, *t*, to travel 100 m is:

$$t = \frac{100\ m}{v} = \frac{100\ m}{0.067\ m/d} = 1490\ d = 4.09\ years$$

This result is indicative of the slow movement of most groundwaters.

3. Limitations of Darcy's law

Darcy's law indicates a linear relationship between the flow rate and the gradient in the piezometric head measured in the direction of flow. This linear relationship is the same as for laminar flow in

[8] Domenico and Schwartz, 1998.

pipes under low Reynolds number and is symptomatic of the dominance of viscous forces as water flows through the pore spaces. Flow in porous media is typically laminar under natural conditions.

As the Reynolds number increases, viscous forces become less dominant, the flow rate deviates from being linearly proportional to the piezometric head gradient, and the relationship becomes nonlinear.

A characteristic Reynolds number, R_e [dimensionless], can be defined in terms of the specific discharge, q [LT^{-1}]; pore scale, d [L]; and kinematic viscosity of water, ν [L^2T^{-1}], by the relation:

$$R_e = \frac{q\,d}{\nu} \tag{11}$$

In unconsolidated formations, d is typically estimated by the 10-percentile grain size, d_{10}, which is then referred to as the effective grain size.

Experiments indicate that deviations from Darcy's law begin to occur for values of Re > 1, but serious deviations do not occur up to Re = 10. [9]

Nonlinear flow transition from being laminar to turbulent is in the range 10 < Re < 100, and fully turbulent flows typically occur when Re > 100.

Nonlinear flow conditions are routinely found near large water supply wells and in fractured rock formations[10].

A variety of empirical and semiempirical equations have been suggested for describing nonlinear flows (Bear, 1972).

EXAMPLE 2

A sand aquifer has a 10-percentile particle size of 0.4 mm and an effective porosity of 0.3. The temperature of the water in the aquifer is 20◦C.

Estimate the range of seepage velocities for which Darcy's law is valid?

Solution

Darcy's law can be taken to be valid when Re < 10,

[9] Ahmed and Sunada, 1969 and Bear, 1972.
[10] Worthington and Gunn, 2009.

$$\frac{q\,d}{\nu} < 10$$

which can be put in the form:

$$q < \frac{10\,\nu}{d}$$

The seepage velocity, v, is:

$$v = \frac{q}{n_e}$$

or

$$q = v\,n_e$$

substituting

$$v\,n_e < \frac{10\;\nu}{d}$$

$$v < \frac{10\;\nu}{d\,n_e}$$

From the given data,
ne = 0.3,
$d \simeq d_{10}$ = 0.4 mm = 4 * 10^{-4} m,
and <u>at 20◦C:</u> ν = 1.00 * 10^{-6} m^2/s.

$$v < \frac{10\,(1*10^{-6})}{0.3\,(4*\,10^{-4})} < 0.0833 \text{ m/s} < 7200 \text{ m/d}$$

Darcy's law can be applied in the aquifer whenever the seepage velocity is less than 7200 m/d.

4. <u>Hydraulic Conductivity</u>

Hydraulic conductivity is the property of a porous medium that describes the relationship between the specific discharge and the gradient in the piezometric head.

Accurate measurement and characterization of the hydraulic conductivity are essential to the quantification of flow in porous media.

<u>Empirical formulae</u>

The hydraulic conductivity, *K*, that appears in Darcy's law is a function of both the fluid properties and the geometry of the solid matrix.

This point is obvious considering the water in Darcy's experiment being replaced by oil. There would be less flow than for water under the same piezometric gradient, thereby indicating a smaller hydraulic conductivity for oil in the sand than for water.

Also, if clay were used instead of sand in Darcy's experiment, then there would be less flow than for sand under the same hydraulic gradient, indicating a smaller hydraulic conductivity for water in clay than for water in the sand.

The functional relationship between the hydraulic conductivity and the fluid and solid-matrix properties can be extracted using dimensional analysis.

If the fluid properties are characterized by the specific weight, γ [FL^{-3}], and dynamic viscosity, μ [$FL^{-2}T$], while the solid matrix is characterized by effective particle size, *de* [L], then the hydraulic conductivity, K [LT^{-1}], is related to the fluid and the solid-matrix properties applying the Buckingham pi theorem.

The size of the pore openings in a granular porous medium depends on both the effective particle size, *de*, and the degree of compactness of the particles as measured by the porosity, *n*. <u>However:</u>

$$K = \{\ k\ \} * \frac{g}{\nu} = \{\propto *\ \varphi(n) *\ d_e^2\} * \frac{g}{\nu} \qquad (12)$$

<u>Where:</u>

k is called the *intrinsic permeability* and depends only on the structure of the solid matrix,
ν is the kinematic viscosity,
α is the structure coefficient,
$\varphi(n)$ is the porosity function.

The intrinsic permeability, *k*, is sometimes expressed in darcy, where 1 darcy is equal to $0.987*10^{-12}$ m^2.
The intrinsic permeability should not be confused with the "coefficient of permeability," a term used by geotechnical engineers to refer to hydraulic conductivity.

Many empirical formulae have been developed to predict the hydraulic conductivity, *K*, from grain-size analyses, as illustrated in Table 3. These empirical formulae are all in the form of Equation 12 and only differ in the estimation of α, $\varphi(n)$, and *de*.
The parameters in these equations are derived from particle-size (sieve) analyses and are defined as follows: d_{10}, d_{17}, d_{20} are the 10-, 17-, and 20-percentile particle sizes respectively, *Uc* is the uniformity coefficient and <u>is defined as:</u>

$$U_c = \frac{d_{60}}{d_{10}} \tag{13}$$

where d_{60} is the 60-percentile particle size.

Table 3. Parameters to Estimate Hydraulic Conductivity K from Grain-size Distribution

Name of Formula	Structure Coefficient, α	Porosity Function, φ(n)	Effective Grain size, d_e	Restrictions
Hazen	$6 * 10^{-4}$	1 + 10 (n – 0.26)	d_{10}	0.1mm < de < 3 mm Uc < 5
Slichter	$1 * 10^{-2}$	$n^{3.287}$	d_{10}	0.01mm<de<5mm

In cases where only the grain-size distribution is known, the porosity, *n*, can be estimated from the uniformity coefficient, *Uc*, using the relation[11]:

$$n = 0.255(1 + 0.83^{Uc}) \tag{14}$$

EXAMPLE 3

Laboratory analyses of an aquifer material indicate a porosity of 0.40, a water temperature of 20∘C, and a grain-size distribution as follows:

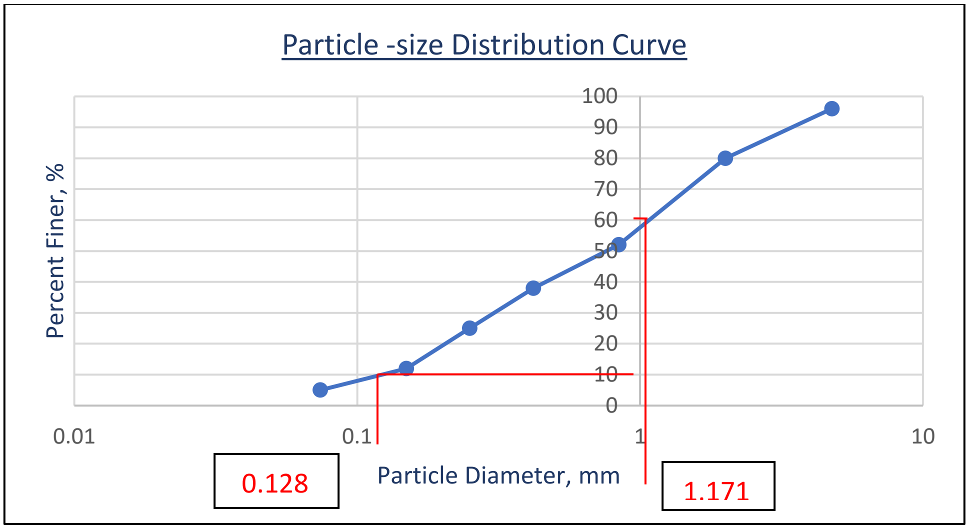
Sieve Number	Grain size, mm	Percent finer, %
4	4.760	96
10	2.000	80
20	0.840	52
40	0.420	38
60	0.250	25
100	0.149	12
200	0.074	5

Estimate the hydraulic conductivity of the aquifer material using empirical equations and compare the results?

Solution

From the given data, *n* = 0.40, and the grain-size distribution is:

[11] Vukovi´c and Soro, 1992.



d_{10} = 0.128 mm, d_{60} = 1.171 mm, and $Uc = d_{60}/d_{10}$ = 1.171/0.128 = 9.1

For water at 20◦C, $\nu = 1.004*10^{-6}$ m²/s, and the hydraulic conductivity, K, (Equation 12) is:

$$K = \{ k \} * \frac{g}{\nu} = \{\propto * \ \varphi(n) * \ d_e^2\} * \frac{g}{\nu}$$

$$K = \{\propto * \ \varphi(n) * \ d_e^2\} * \frac{9.81}{1.004 * 10^{-6}} = 9.77 * 10^6 * \propto * \ \varphi(n) * d_e^2$$

where K is in m/s if de is expressed in meters.

Name of Formula	Structure coefficient, α	Porosity function, φ(n)	Effective grain size, d_e, m	Restrictions	K, m/d
Hazen	$6 * 10^{-4}$	1 + 10 (n – 0.26) =1+10(0.4-0.26) =2.4	$d_{10} = 0.128*10^{-3}$	0.1mm < de < 3 mm Uc < 5 Uc = 9.1 > 5	19.9
Slichter	$1 * 10^{-2}$	$n^{3.287} = 0.4^{3.287}$ = 0.0492	$d_{10} = 0.128*10^{-3}$	0.01mm<de<5mm	6.8

The hydraulic conductivity is estimated using the empirical formulae to be 6.8 m/d when restrictions on applying the empirical formulae are respected.

For sieve analysis, refer to this reference:

http://www.ce.memphis.edu/1101/notes/filtration/filtration-2.html

Figure 7. Set of sieves in a sieve shaker

Table 4. The U.S. Standard sieve sizes

Sieve Number	Opening (mm)
4	4.750
6	3.350
8	2.360
10	2.000
16	1.180
20	0.850
30	0.600
40	0.425
50	0.300
60	0.250
80	0.180
100	0.150
140	0.106
170	0.088
200	0.075
270	0.053

Table 5 illustrates a classification of hydraulic conductivities, K.

Table 5. Classification of Hydraulic Conductivity, *K* – [USBR, 1977]

K, m/d	Classification	Unconsolidated Deposits	Consolidated Rocks
> 1000	Very high	Clean gravel	Vesicular and scoriaceous basalt (البازلت الحويصلي والصدفي) and cavernous limestone (الحجر الجيري الكهفي) and dolomite
10 - 1000	High	Clean sand, and sand and gravel	Clean sandstone and fractured igneous and metamorphic rocks
0.01 - 10	Moderate	Fine sand	Laminated sandstone, shale, and mudstone
0.0001 - 0.01	Low	Silt, clay, and a mixture of sand, silt, and clay	Massive igneous and metamorphic rocks
< 0.0001	Very low	Massive clay	---

5. Flow in the Unsaturated Zone

The fields of surface-water hydrology and groundwater hydrology are linked by processes that occur within the unsaturated (aeration) zone, as shown in Figure 8. Understanding the movement of water in the unsaturated zone is important in describing the infiltration process, the extraction of water for the plant, and the movement of water and contaminants from the ground surface into the saturated zone of an aquifer. Water in the soil-water zone is called *soil water*.

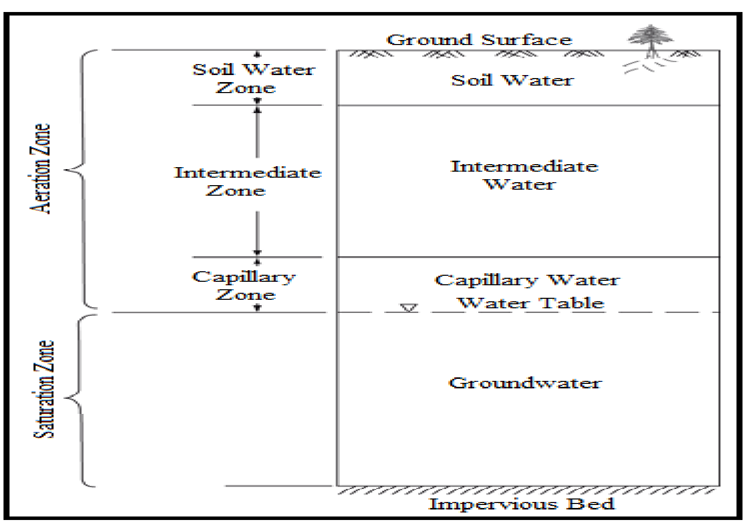


Figure 8. Aeration and Saturation Zones

Transpiration, evaporation, and gravity drainage are the main processes that control the moisture content within the soil-water zone. Below the soil-water zone, evaporation and transpiration are negligible.

Within the capillary zone, which is sometimes called the *capillary fringe*, the water rises from the water table through the void spaces in much the same way that water rises in a capillary tube.

To illustrate the dynamics of water movement in the capillary zone, consider the capillary tube shown in Figure 9, with the *surface tension* between the capillary-tube material and water given by σ. Equilibrium of the column of water within the capillary tube requires that the weight of the water column be supported by the surface tension force between the tube material and the water.

Therefore:

$$F_W = F_{S.T} \tag{12}$$

$$\gamma \frac{\pi D^2}{4} h_c = \pi D \sigma \cos \vartheta \tag{13}$$

where γ is the specific weight of water, *D* is the diameter of the capillary tube, *hc* is the capillary rise, σ is surface tension (force per unit length), and θ is the *contact angle* between the water and the tube.

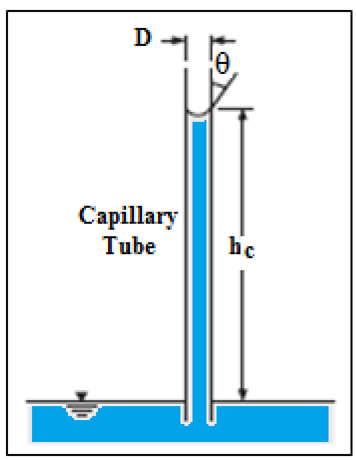


Figure 9. Capillary Rise

Rearranging Equation yields the following expression for the capillary rise, *hc*, as:

$$h_c = \frac{4 \sigma \cos \theta}{\gamma D} \tag{14}$$

This relationship shows that the capillary rise is inversely proportional to the diameter of the tube.

Since the pressure distribution within the capillary tube is hydrostatic, the pressure, p, at any height z above the water surface is given by:

$$p_c = -\gamma z \tag{15}$$

where the pressure is negative and therefore below atmospheric pressure.

Extending the behavior of capillary tubes to pore spaces above the saturated zone in groundwater, it is clear why the pressure is negative in the pore water above the water table.

In fine media such as silts and clays, where the diameters of the pores are small, there are large capillary rises. Conversely, in coarse materials such as sand and gravel, the pores have large diameters, and the capillary rise is generally small.
The capillary rises in several samples of unconsolidated materials having similar porosity ($n \simeq 0.41$) are given in Table 6.

Table 6. Capillary Rise in Unconsolidated Materials – [Lohman, 1979]

Material	Grain size, mm	Capillary rise, cm
Fine gravel	2 – 5	2.5
Very coarse sand	1 – 2	6.5
Coarse sand	0.5 – 1.0	13.5
Medium sand	0.2 – 0.5	24.6
Fine sand	0.1 – 0.2	42.8
Silt	0.05 – 0.10	105.5
Fine silt	0.02 – 0.05	200.0

Although capillary rise through porous media is very much like the rise through capillary tubes, porous media differ from capillary tubes in that the pores are irregular, vary in size according to the gradation of the solid matrix, and sometimes contain “dead ends”.

Consequently, the moisture content in the porous medium, θc, decreases with distance above the water table and is defined by the relation:

$$\theta_c = \frac{water\ volume\ in\ soil\ sample}{sample\ volume} \tag{16}$$

The moisture content continues to decrease with distance above the water table until the continuity of the capillary rise is broken. Then, the distribution of water is no longer similar to the rise in a capillary tube but becomes discontinuous and held by the solid matrix.

The distribution of moisture capacity within the unsaturated zone is illustrated in Figure 10, which is commonly called the *retention curve*. Its exact shape depends on several factors, with pore-size distribution being the most important.

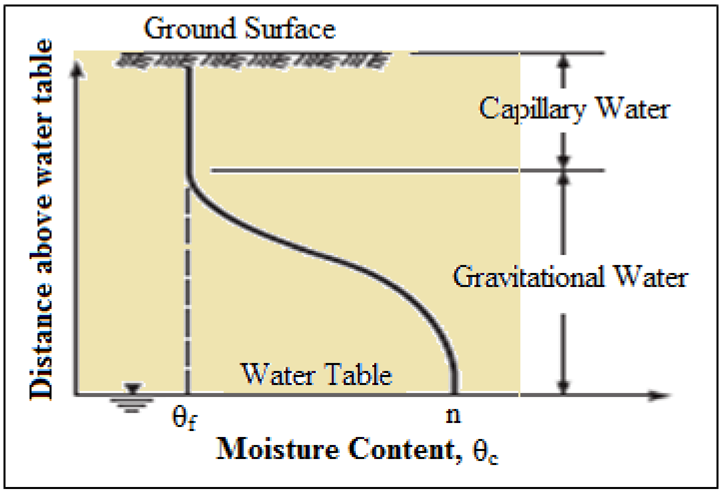


Figure 10. Retention Curve

Within the soil-water zone and intermediate zone, the moisture capacity is determined by surface tension between the soil and water, and the maximum (equilibrium) moisture content is equal to the *field capacity*, θf.

Within the capillary zone, the pressure distribution in the pore water is described by Equation 15, where z is the elevation above the water table.

Water that is held discontinuously at the field capacity is called *capillary water*, while water that is continuously drawn from the water table is called *gravitational water*.

The maximum moisture content in the retention curve is equal to the porosity, n.

When the moisture content is reduced to the *wilting point*, the water is held by the soil so tightly that plants cannot extract any more water, and only evaporation can further reduce the moisture content.

The conventional measure of the wilting point is the water content corresponding to a capillary suction of 1.5 MPa.

The soil can ultimately become dry as all the capillary water is removed by evaporation.
At this point, a small amount of water called *hygroscopic water* is still held by the soil, and this water can be removed by oven drying the soil at a temperature of 105◦C.

However, the range of moisture contents is illustrated in Figure 11.

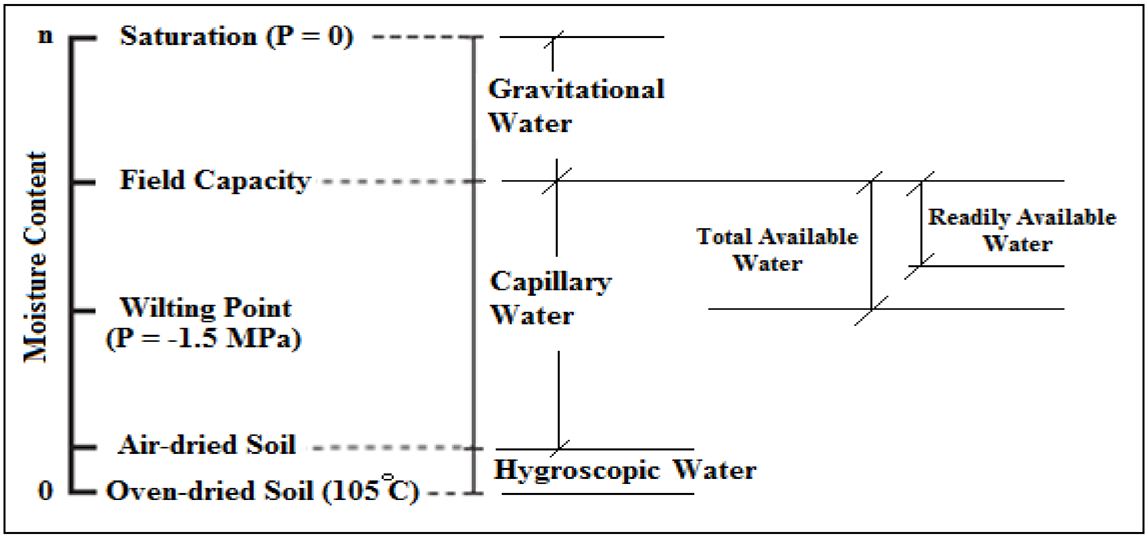


Figure 11. Range of Moisture Contents in Porous Media

Since gravitational water is continuously connected to the water table, it is replenished by capillary rise, and therefore significant reductions in moisture content do not necessarily result from the extraction of gravitational water.

The *specific storage*, *Ss*, of a porous medium is defined as the volume of water released from storage per unit volume of the porous medium per unit decline in the piezometric head. Typical values of *Ss* in unconsolidated materials are in the range of $10^{-4} - 10^{-9}$ m^{-1} (Roscoe Moss Company, 1990; Cheng, 2000).

The soil is *isotropic* when the hydraulic conductivity does not depend on the flow direction. In cases where the hydraulic conductivity depends on the flow direction, the porous medium is called *anisotropic*.

The *specific yield*, *Sy*, of an unconfined aquifer is defined as the volume of water released from storage per unit (plan) area per unit decline in the phreatic surface (water table). Representative values of the specific yield for several aquifer materials are given in previous Tables 1 and 2.

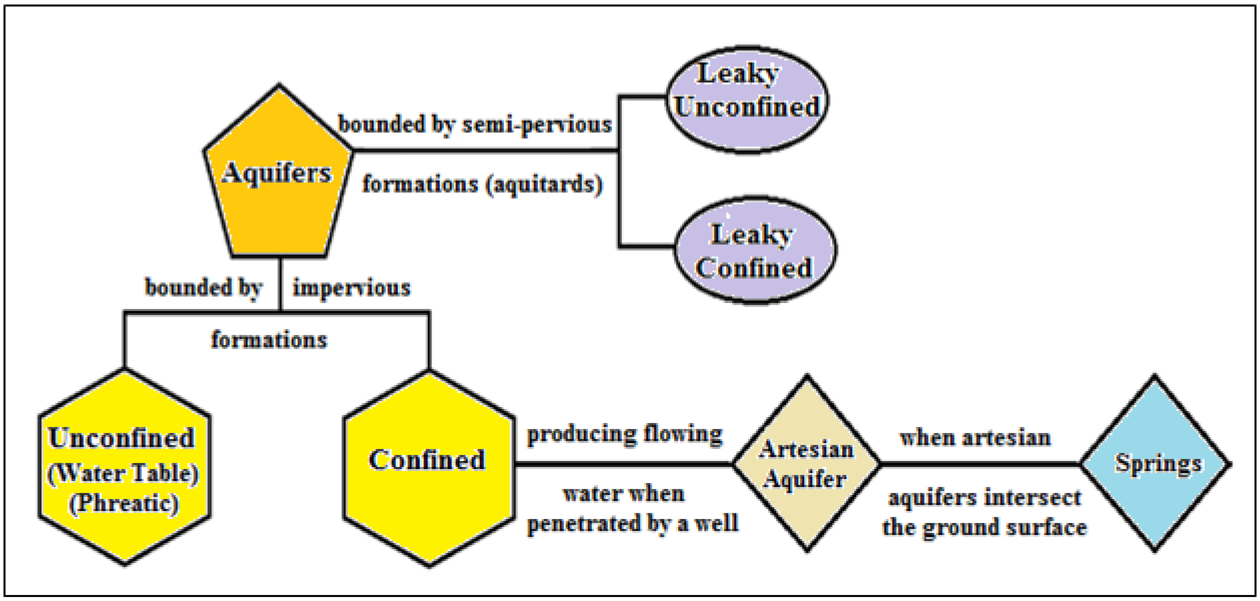
SUMMARY

Aquifers are geologic formations containing water that can be withdrawn in significant amounts.

Aquicludes contain water but are incapable of transmitting it in significant quantities (clay layers). Aquicludes can be taken as impervious formations.

Aquifuges neither contain nor transmit water (solid rocks).

Aquitards are semi-pervious formations that are significantly less permeable than the aquifer but are not impervious.



$$n = \frac{voids\ volume}{sample\ volume}$$	$n < 0.05$ Small
	$0.05 < n < 0.20$ Medium
	$n > 0.20$ Large

$Q = K\,A\,\frac{\varphi_1 - \varphi_2}{L}$	&	$J = \frac{\varphi_2 - \varphi_1}{L}$	∴	$q = \frac{Q}{A} = -K\,J$

q = specific discharge = filtration velocity = Darcy's velocity

v: seepage velocity $v = \frac{Q}{A_p}$	&	$q = \frac{Q}{A}$	∴	$Q = v\,A_p = q\,A$	➡	$v = q * \frac{A}{A_p}$

$n_e = \frac{A_e}{A}$	➡	Unconsolidated $n_e \simeq n$	&	Consolidated ne < n

Taking $A_p = A_e$	➡	$v = q * \frac{A}{A_e}$	➡	$v = \frac{q}{n_e} = -\frac{K}{n_e}J$

$\boldsymbol{R_e = \frac{q\,d}{\nu}}$	Re < 100	Laminar flow
	10 < Re < 100	Transition flow
	Re > 100	Turbulent flow

In unconsolidated formations, *d* is the 10-percentile grain size, d_{10}, (the effective grain size).

Flow in porous media is typically laminar under natural conditions.

Nonlinear flow is found near large water supply wells and in fractured rock formations.

$\boldsymbol{K = \{\,k\,\} * \frac{g}{\nu} = \{\propto * \;\varphi(n) * \;d_e^2\} * \frac{g}{\nu}}$	k is ***intrinsic permeability*** and depends only on the structure of the solid matrix.
	It is sometimes expressed in darcy, where 1 darcy is equal to $0.987*10^{-12}$ m^2

$U_c = \frac{d_{60}}{d_{10}}$	$n = 0.255(1 + 0.83^{Uc})$	$h_c = \frac{4\,\sigma\cos\theta}{\gamma\,D}$

$p_c = -\,\gamma\,z$	In the capillary zone, the pressure is negative in the pore water above the water table.

$$\theta_c = \frac{volume\ of\ water\ in\ soil\ sample}{sample\ volume}$$

θ_c decreases with distance above the water table.

The distribution of moisture capacity within the unsaturated zone is called the retention curve.

The field capacity is the maximum moisture content determined by surface tension between the soil and water within the soil-water zone and intermediate zone.

Capillary water is water held discontinuously at the field capacity.

Gravitational water is water continuously drawn from the water table.

Hygroscopic water is a small amount of water still held by the soil and can be removed by oven drying the soil at a temperature of 105°C.

Pendular water is the water clinging to soil particles and not drained under the force of gravity.

The maximum moisture content in the retention curve is equal to the porosity, n.

The specific storage (Ss) [L^{-1}] is the volume of water released from storage per unit volume of the porous medium per unit decline in the piezometric head.

The soil is isotropic when the hydraulic conductivity does not depend on the flow direction.

The soil is anisotropic when the hydraulic conductivity depends on the flow direction.

The specific yield (Sy) of an unconfined aquifer is the volume of water released from storage per unit plan area per unit decline in the phreatic surface (water table).

CHAPTER 2

Fundamentals of Groundwater Hydrology II: Applications

1. Introduction

Most engineering applications of groundwater hydrology involve specific solutions to the groundwater flow equation, and these solutions can be either numerical or analytic.

Numerical models of groundwater flow typically solve the three-dimensional flow equation given by the following general Equation (1) and can easily accommodate complex initial and boundary conditions along with complex hydraulic conductivity and specific storage distributions within an aquifer.

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial \varphi}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_{yy}\frac{\partial \varphi}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_{zz}\frac{\partial \varphi}{\partial z}\right)=S_s\frac{\partial \varphi}{\partial t} \qquad (1)$$

Analytic models typically solve the two-dimensional (Dupuit) approximations to the groundwater flow equations and are appropriate whenever the characteristics of the porous formation, the boundary, and the initial conditions are particularly simple.
Analytic models are commonly applied for matching observed aquifer responses with analytic solutions to determine aquifer properties.

2. Steady-State Solutions

Steady-state conditions occur when all boundary conditions are constant in time.

Such conditions seldom occur in reality; however, in those cases where the boundary conditions change slowly in time or long-term average flow conditions are being studied, steady-state solutions can be applied as reasonable approximations.

2.1. Unconfined Flow between Two Reservoirs

Figure 1 shows the case of unconfined flow between two reservoirs.

The flow is from a reservoir with water depth h_L to a reservoir with water depth h_R, through a distance L of a stratified phreatic aquifer with hydraulic conductivities K_1 and K_2 over thicknesses b_1 and b_2, respectively.

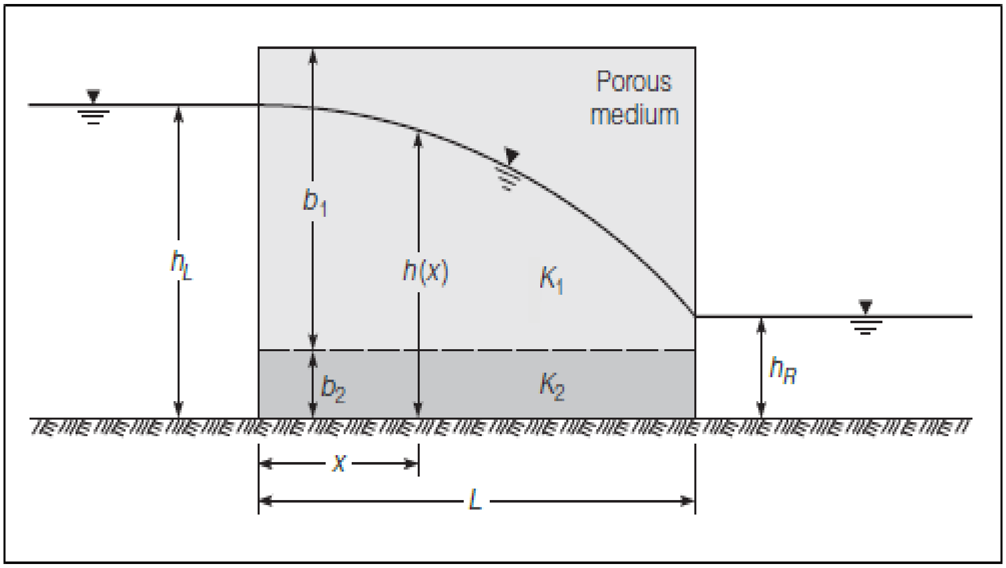


Figure 1. Unconfined Flow between Two Reservoirs

According to the Dupuit approximation, applying the boundary conditions on the governing flow equation (Equation 14.87 in the Reference), the following equations are deduced:

$$\frac{K_1}{2}\,h^2 + (K_2 - K_1)\,b_2\,h = C_1 x + C_2 \qquad (2)$$

$$C_1 = \frac{K_1}{2\,L}\,(h_R^2 - h_L^2) + (K_2 - K_1)\,\frac{b_2}{L}\,(h_R - h_L) \qquad (3)$$

$$C_2 = \frac{K_1}{2}\,(h_L^2) + (K_2 - K_1)\,b_2\,h_L \qquad (4)$$

The complete solution to the problem of unconfined flow between two reservoirs is given by Equation 2, where the constants C_1 and C_2 are given by Equations 3 and 4.

Once the distribution of saturated-zone thickness, *h(x)*, is calculated, the flow per unit aquifer width between the two reservoirs, Q [L^2T^{-1}], can be calculated directly from Darcy's law and the Dupuit approximation using the simple relation:

$$Q = -\,C_1 \qquad (5)$$

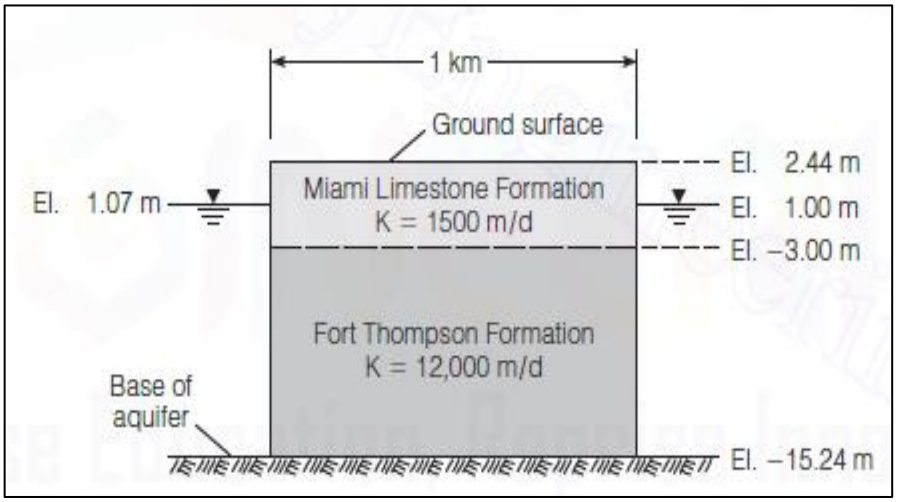
The approach used here can be extended to cases where there are more than two layers, and to cases where a recharge, *N*, is present.

EXAMPLE 1

The Biscayne aquifer, one of the most permeable aquifers in the world, consists principally of two layers: the Miami Limestone Formation and the Fort Thomson Formation. In one area, the Miami Limestone Formation extends from the ground surface at elevation 2.44 m to elevation −3.00 m, and the Fort Thomson Formation extends from elevation −3.00 m to elevation −15.24 m. The hydraulic conductivity of the Miami Limestone Formation can be taken as 1500 m/d, and the hydraulic conductivity of the Fort Thompson Formation is 12,000 m/d.

1) Calculate the shape of the phreatic surface?
2) For two fully penetrating canals 1 km apart with water-surface elevations of 1.07 m and 1.00 m, find the flow rate between the two canals?

Solution



A schematic diagram of the Biscayne aquifer between two fully penetrating canals is shown in the Figure. The phreatic surface is described by Equations 2, 3, and 4, where:

h_L = 1.07 m − (−15.24 m) = 16.31 m, h_R = 1.00 m − (−15.24 m) = 16.24 m,

K_1 = 1500 m/d, K_2 = 12,000 m/d,

b_2 = −3.00 m − (−15.24 m) = 12.24 m, and L = 1000 m.

$750\, h^2 + 128{,}520\, h = C_1 x + C_2$ where: $C_1 = -10.7$ m^2/d, $C_2 = 2{,}296{,}000$ m^3/d

Combining these results gives the following equation for the phreatic surface:

$$h^2 + 171.4\, h = -0.01428\, x + 3061$$

The flow rate, Q, between the two reservoirs is given by Equation 5, where:

$$Q = -C_1 = -(-10.7 \text{ m}^2/\text{d}) = 10.7 \text{ m}^2/\text{d}$$

2.2. Wells in a Confined Aquifer

In isotropic[12] and homogeneous confined aquifers, as shown in Figure 2, the drawdown surface surrounding the well (the *cone of depression*) is given by Thiem equation:

$$T = \frac{Q_w}{2\,\pi\,(S_1 - S_2)} \ln \frac{r_2}{r_1} \qquad (6)$$

Where: Qw, the pumping rate.

S_1 and S_2, drawdowns at two monitoring wells located at $r = r_1$ and $r = r_2$, respectively.

T, the transmissivity of the aquifer. [13]

b, the aquifer thickness.

K, the hydraulic conductivity of the aquifer.

$$T = K * b \qquad (7)$$

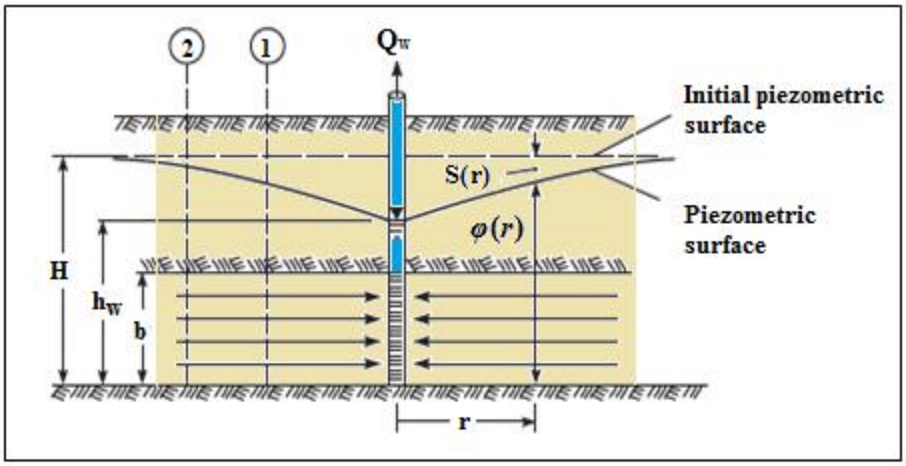


Figure 2. Fully Penetrating Well in a Confined Aquifer

It must be noted that the piezometric head, φ, is radially symmetric and independent of time.

There is no recharge of the confined aquifer.

[12] When the hydraulic conductivity does not depend on the flow direction, the porous medium is called *isotropic*.

[13] It is the rate at which groundwater flows horizontally through an aquifer section of unit width under a unit hydraulic gradient.

EXAMPLE 2

An aquifer pump test was conducted in a confined aquifer where the initial piezometric surface was at elevation 14.385 m, and well logs indicate that the thickness of the aquifer is 25 m. The well was pumped at 31.54 L/s, and after 1 day the piezometric levels at 50 m and 100 m from the pumping well were measured as 13.585 m and 14.015 m, respectively.

Assuming steady-state conditions, estimate the transmissivity and hydraulic conductivity of the aquifer?

Solution

Q_w = 31.54 L/s = 2,725 m^3/d,
r_1 = 50 m, r_2 = 100 m,
S_1 = 14.385 m − 13.585 m = 0.800 m,
S_2 = 14.385 m − 14.015 m = 0.370 m.

$$T = \frac{Q_w}{2\,\pi\,(S_1 - S_2)} \ln\frac{r_2}{r_1}$$

$$T = \frac{2725}{2\,\pi\,(0.800 - 0.370)} \ln\frac{100}{50} = 699 \text{ m}^2/\text{d}$$

$$T = K * b$$

$$K = \frac{T}{b} = \frac{699}{25} \sim 28\, m/d$$

Therefore, the results of the aquifer pump test indicate that the transmissivity of the aquifer is 699 m^2/d, and the hydraulic conductivity is 28 m/d.

Aquifers with transmissivity values less than 10 m^2/d can supply enough water only for domestic wells and other low-yield uses [14].

A key assumption that is implicit in the Thiem equation is that the aquifer is homogeneous.

There are theoretical methods to estimate the statistics of the spatial distribution of transmissivity from aquifer tests at a single location [15].

[14] Lehr et al., 1988.
[15] e.g., Schneider and Attinger, 2008.

2.3. <u>Wells in an Unconfined Aquifer</u>

In isotropic and homogeneous unconfined aquifers, as shown in Figure 3, the employed equation is Dupuit equation:

$$T = \frac{Q_W}{2\,\pi\,(S\prime_1 - S\prime_2)} \ln\frac{r_2}{r_1} \qquad (8)$$

Where: Qw, the pumping rate.

S_1 and S_2, drawdowns at two monitoring wells located at $r = r_1$ and $r = r_2$, respectively.

T, the transmissivity of the aquifer.

H, the aquifer saturated thickness.

K, the hydraulic conductivity of the aquifer.

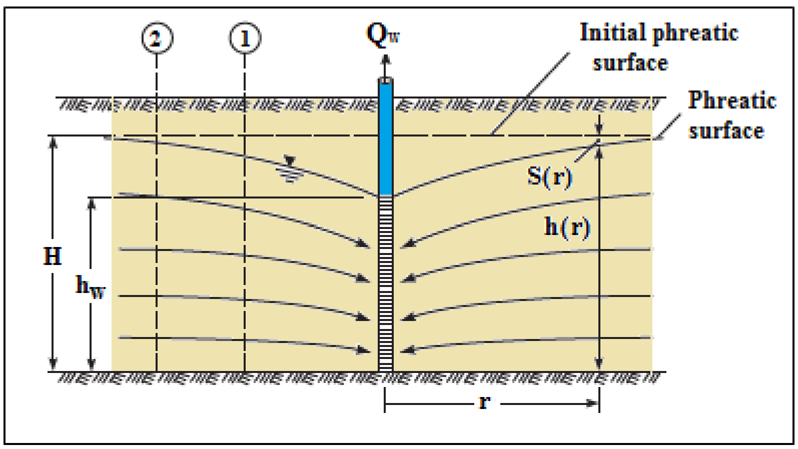


Figure 3. Fully Penetrating Well in an Unconfined Aquifer

$$T = K * H \qquad (9)$$

S'_1 and S'_2, modified drawdowns, are given by:

$$S_1' = S_1 - \frac{S_1^2}{2\,H} \qquad (10)$$

$$S_2' = S_2 - \frac{S_2^2}{2\,H} \qquad (11)$$

The equation of an unconfined aquifer is the same result that was derived for a confined aquifer except that the modified drawdowns are used.

It should be noted, however, that in the common case where the drawdowns are small compared with the aquifer saturated thickness, <u>or specifically:</u>

$$S_1^2 \ll 2\,H$$
$$S_2^2 \ll 2\,H$$

then, the modified drawdowns, S'_1 and S'_2, can be replaced by the actual drawdowns, S_1 and S_2.

EXAMPLE 3

An aquifer pump test has been conducted in an unconfined aquifer of saturated thickness 15 m. The well was pumped at a rate of 100 L/s, and the drawdowns at 50 m and 100 m from the pumping well after 1 day of pumping were 0.412 m and 0.251 m, respectively.

For steady-state, estimate the transmissivity and hydraulic conductivity of the aquifer?

Solution

Q_w = 100 L/s = 8,640 m^3/d, r_1 = 50 m, r_2 = 100 m, S_1=0.412 m, S_2=0.251 m, H=15m

$$S'_1 = S_1 - \frac{S_1^2}{2\,H}$$

$$S'_1 = 0.412 - \frac{0.412^2}{2 * 15} = 0.406\,m$$

$$S'_2 = S_2 - \frac{S_2^2}{2\,H}$$

$$S'_2 = 0.251 - \frac{0.251^2}{2 * 15} = 0.249\,m$$

$$T = \frac{Q_w}{2\,\pi\,(S'_1 - S'_2)} \ln\frac{r_2}{r_1}$$

$$T = \frac{8640}{2\,\pi\,(0.406 - 0.249)} \ln\frac{100}{50} = 6070\,m^2/d$$

$T = K * H$ $\qquad K = \frac{T}{H} = \frac{6070}{15} \sim 405\,m/d$

<u>Therefore,</u> the transmissivity and hydraulic conductivity of the aquifer are estimated from the aquifer pump-test data to be 6070 m^2/d and 405 m/d, respectively.

3. Saltwater Intrusion

In coastal aquifers, a transition region exists where the water in the aquifer changes from freshwater to saltwater. Since saltwater is denser than freshwater, the saltwater tends to form a wedge beneath the freshwater, as shown in Figure 4 for the case of an unconfined aquifer.

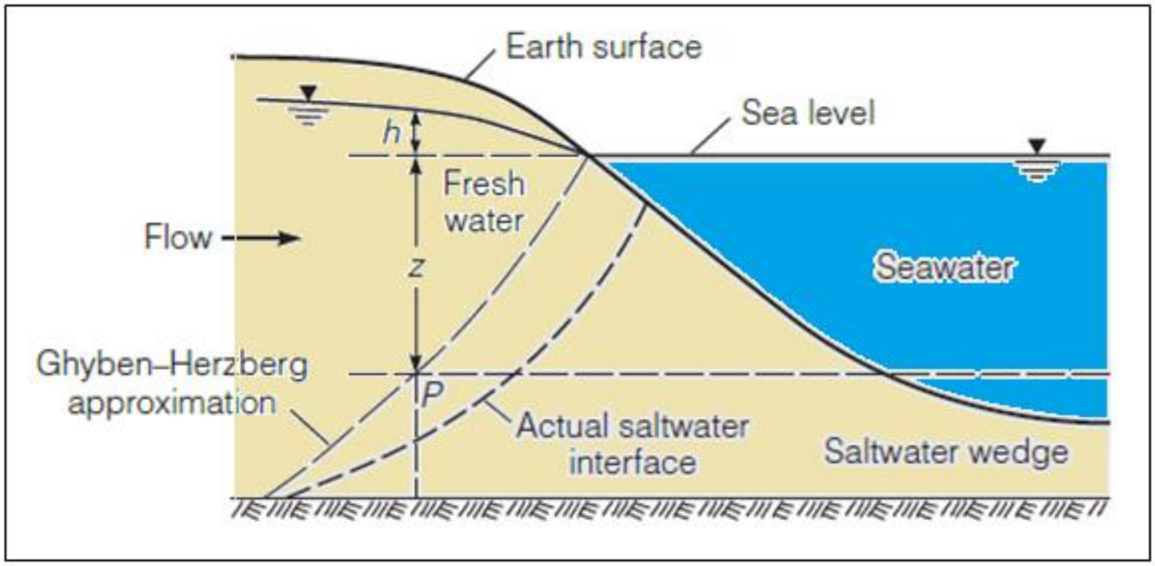


Figure 4. Saltwater Interface in a Coastal Aquifer

This illustration is somewhat idealized since there is not a sharp interface between freshwater and saltwater zones, but rather a "blurred" interface resulting from diffusion and mixing caused by the relative movement of the freshwater and saltwater.

This blurred interface between the freshwater and saltwater zones is sometimes referred to as the *zone of salinity transition*[16] or simply the *transition zone*[17].

The saltwater interface is commonly taken as the 10,000 mg/L iso-salinity lines since water with higher salinity is considered unsuitable for human use.

The volume of water below the saltwater interface is called the *saltwater wedge*.

Seawater within the saltwater wedge is not static, as is often assumed, but flows inland along the base of the aquifer, mixes with the seaward flowing freshwater, and discharges to the sea.

[16] Prieto and Destouni, 2005.
[17] Motz and Sedighi, 2009.

Consequently, the groundwater discharged to the sea consists of both freshwater and recycled seawater.

The relative movement between the freshwater and saltwater zones is usually associated with mean groundwater flow toward the coast, tides, and temporal variations in aquifer stresses.

The thickness of the freshwater zone can range from a few meters to over 100 meters.

The intrusion of saltwater into coastal aquifers is generally of concern because of the associated drop in groundwater quality.

Since the recommended maximum contaminant level (MCL) for chloride in drinking water is 250 mg/L and a typical chloride level in seawater is 14,000 mg/L, mixing more than 1.8% seawater with non-saline water makes the mixture non-potable.

This percentage is even less if the freshwater contains a non-zero chloride concentration.

The location of the freshwater–saltwater interface is usually tracked in coastal aquifers using monitoring wells.

The impacts of saltwater intrusion in coastal aquifers are likely to increase as sea levels continue to rise in the 21st century. Reported average rates of sea-level rise are on the order of 2 mm/yr (0.08 in/yr) for the 20th century[18] and on the order of 3 mm/yr (0.12 in/yr) in the years 1992–2010[19].

Since approximately 60% of the world's population lives within 30 km (20 mi) of the shoreline[20] and groundwater is a key water supply source in these areas, saltwater intrusion is an ever-present concern for much of the world.

An approximate method to determine the location of the saltwater interface is called the *Ghyben–Herzberg approximation*[21].

Under this approximation, the pressure distribution is assumed to be hydrostatic within any vertical section of the aquifer, which assumes that the streamlines are horizontal.

Under this assumption, the hydrostatic pressure at point (*P)* in Figure 4 can be calculated from either the freshwater head or the saltwater head.

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[18] Douglas, 1997.
[19] Nicholls and Cazenave, 2010.
[20] Loa´ıciga et al., 2012.
[21] W. Badon-Ghyben, 1888 and A. Herzberg, 1901.

γ_f (h + z) = γ_s * z (12)

Where: γ_f is the specific weight of freshwater, γ_s is the specific weight of saltwater, h is the elevation of the water table above sea level, and z is the depth of the saltwater interface below sea level.

$$z = \frac{\gamma_f}{\gamma_s - \gamma_f}\, h = \frac{\rho_f}{\rho_s - \rho_f}\, h = \frac{h}{\epsilon} \quad (13)$$

This is called the *Ghyben–Herzberg equation.*

$$\epsilon = \frac{\rho_s - \rho_f}{\rho_f} \quad (14)$$

Where: ε is called the *buoyancy factor*.

Under typical conditions, ρ_f = 1000 kg/m^3 and ρ_s = 1025 kg/m^3 which yields ε = 0.025, and then:

$$z \approx 40\, h \quad (15)$$

This means that the saltwater interface will typically be found at a distance below sea level equal to 40 times the elevation of the water table above sea level.

Although the factor of 40 is commonly used, under typical densities of freshwater and saltwater, this factor can vary between 33 and 50. [22]

Near the shore, the static assumption used in the Ghyben–Herzberg approximation is less valid and the depth to the interface predicted by the Ghyben–Herzberg approximation tends to be less than the actual depth observed in the field. [23]

In fact, at the shoreline, the Ghyben–Herzberg approximation predicts that the saltwater interface is at sea level, while there must necessarily be a nonzero thickness of freshwater.

Assuming that the flow in the freshwater portion of the aquifer is horizontal and toward the coast, neglecting direct surface recharge (such as from rainfall), and assuming that there is no flow within the saltwater wedge, the flow rate, *Q*, of freshwater toward the coast can be estimated using the Darcy equation as:

$$Q = K\,(h + z)\,\frac{dh}{dx} \quad (16)$$

Where: *K* is the hydraulic conductivity of the aquifer, and *x* is the distance inland from the shoreline.

[22] Werner and Simmons, 2009.
[23] Fitts, 2002.

Equation 16 employs the Dupuit approximation, which assumes horizontal flow and equates the horizontal piezometric head gradient to the slope of the water table.

For boundary conditions ($h = 0$ at $x = 0$ and $h = h_L$ at $x = L$):

$$Q = \frac{K}{2\,\varepsilon\,L}\,h_L^2 \qquad (17)$$

This equation is used to estimate the flow of freshwater toward the coast according to the elevation, h_L, of the water table at a distance L from the coast.

Also, *the water-table profile can be estimated by:*

$$h = h_L\sqrt{\frac{x}{L}} \qquad (18)$$

Since there must be a finite width to accommodate freshwater flow, as illustrated in Figure 5, the finite width, W, of freshwater flow at the coastline was estimated theoretically as: *Glover (1959)*

$$W = \frac{Q}{2\,\varepsilon\,K} \qquad (19)$$

For boundary conditions ($h = 0$ at $x = -W$ and $h = h_L$ at $x = L$):

$$Q = \frac{K}{2\,\varepsilon\,(L-W)}\,h_L^2 \qquad (20)$$

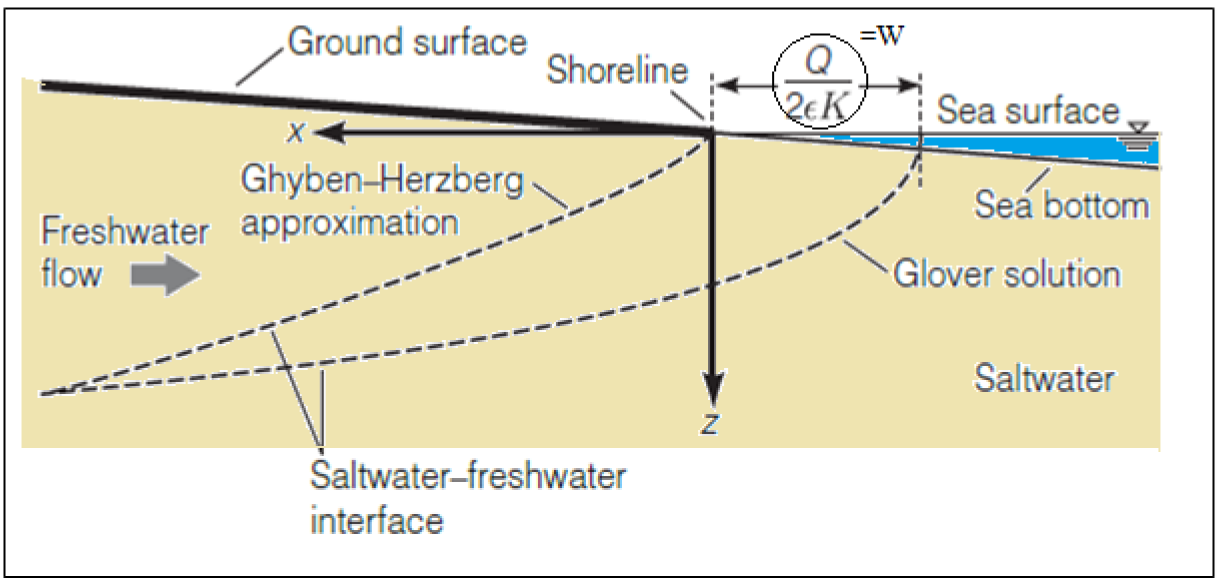


Figure 5. Saltwater Interface at the Shoreline

The depth of the saltwater interface below sea level, z, at any distance, x, from the coastline can be estimated by:

$$z = \sqrt{\frac{2}{\varepsilon K}(Q\,x - C)} \qquad (21)$$

Where: C = 0 corresponds to applying the Ghyben–Herzberg approximation at the coastline, and $C = -QW$ corresponds to applying the more realistic Glover solution at the coastline.

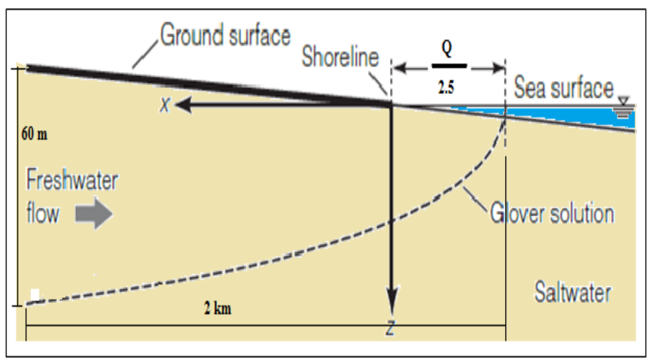
All the relationships presented here neglect the influence of tidal oscillations on the location of the saltwater interface.

EXAMPLE 4

Measurements in a coastal aquifer indicate that the saltwater interface intersects the bottom of the aquifer approximately 2 km from the shoreline. The hydraulic conductivity of the aquifer is 50 m/d and the bottom of the aquifer is 60 m below sea level.

Estimate the freshwater discharge per kilometer of shoreline?

Solution



From the given data:
x = 2 km = 2000 m, K = 50 m/d, and z = 60 m.

Using the Glover solution:

$$z = \sqrt{\frac{2}{\varepsilon K}(Qx - C)} = \sqrt{\frac{2}{\varepsilon K}(Qx + QW)}$$

$$= \sqrt{\frac{2}{\varepsilon K}\left(Qx + \frac{Q^2}{2\,\varepsilon K}\right)}$$

This Equation is a quadratic equation in Q, which can be expressed as:

$$z^2 = \frac{2}{\varepsilon K}\left(Qx + \frac{Q^2}{2\,\varepsilon K}\right)$$

OR $\quad Q^2 + (2\,\varepsilon\,K\,x)\,Q - (\varepsilon\,K\,z)^2 = 0$

The positive root of the quadratic formula of Q is:

$$Q = \frac{-(2\varepsilon Kx) + \sqrt{(2\varepsilon Kx)^2 + 4(\varepsilon\, K\, z)}}{2}$$

$$Q = \varepsilon\, K\, (\sqrt{x^2 + z^2} - x)$$

Assuming ε = 0.025 and substituting the given parameters into the above equation yields:

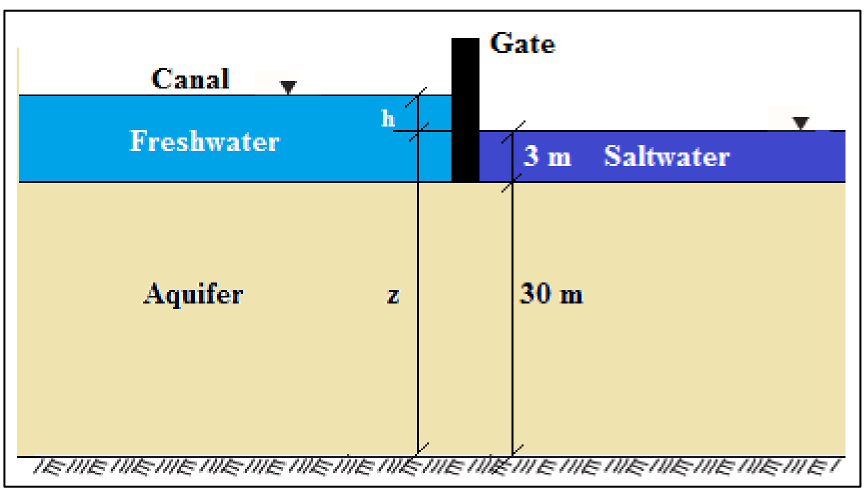
$$Q = \varepsilon\, K \left(\sqrt{x^2 + z^2} - x\right) = 0.025 * 50 \left(\sqrt{2000^2 + 60^2} - 2000\right) = 1.12\, m^2/d$$

*Therefore, the freshwater discharge per kilometer of shoreline is 1.12 * 1,000 = 1,120 (m^3/d)/km*

EXAMPLE 5

Consider the gated canal in a coastal aquifer illustrated in the following Figure. The aquifer thickness below the canal is 30 m, and at high tide, the depth of seawater on the downstream side of the gate is 3 m.

Find the minimum depth of freshwater on the upstream side of the gate that must be maintained to prevent saltwater intrusion?



Solution

The minimum elevation of the freshwater surface at the upstream side of the gate must be sufficient to maintain the saltwater interface at a depth of 33 m below sea level.

According to the Ghyben–Herzberg equation, the height of the freshwater surface above sea level, *h*, is given by: $h = \varepsilon * z$
where ε is the buoyancy factor and *z* is the depth of the interface below sea level.

Taking ε = 0.025 and z = 33 m yields:

$$h = (0.025)(33) = 0.83\text{m}$$

Therefore, the freshwater on the upstream side of the gate must be at least 0.83 m above the sea level on the downstream side of the gate.

Under this condition, the min total depth of freshwater in the canal is:

$$3\text{ m} + 0.83\text{ m} = 3.83\text{ m}$$

Whenever water-supply wells are installed above the saltwater interface, the pumping rate from the wells must be controlled so as not to pull the saltwater up into the well.

The process by which the saltwater interface rises in response to pumping is called *upconing*, as illustrated in Figure 6.

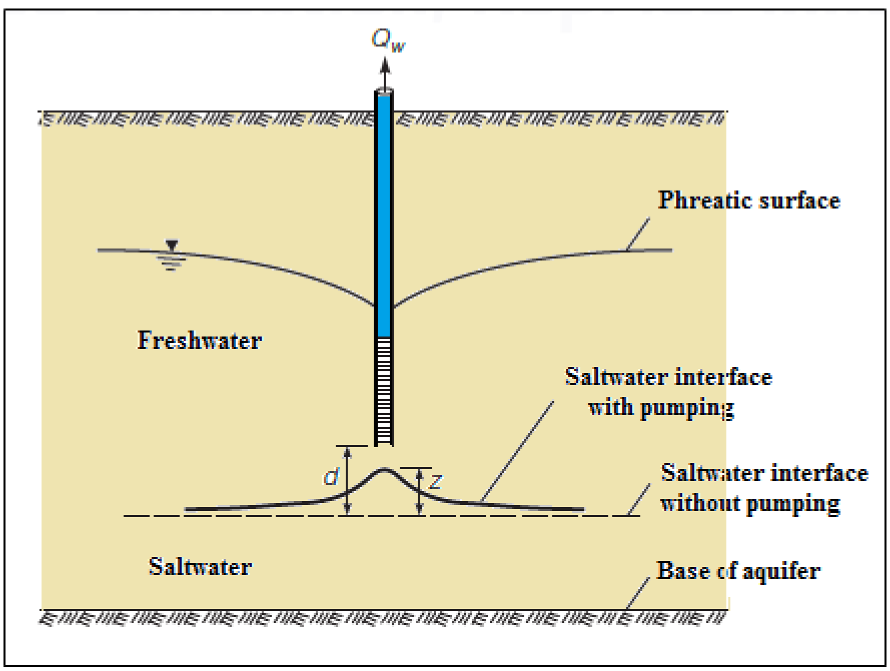


Figure 6. Upconing under a Partially Penetrating Well

Schmorak and Mercado (1969) used equations developed by Dagan and Bear (1968) to propose the following approximation of <u>the rise height, z, of the saltwater interface in response to pumping:</u>

$$z = \frac{Q_w}{2\,\pi\, d\, K_x\, \varepsilon} \qquad (22)$$

Where: Qw is the pumping rate,
d is the depth of the saltwater interface below the well before pumping,
K_x is the horizontal hydraulic conductivity of the aquifer.

This Equation incorporates both the Dupuit and Ghyben–Herzberg approximations.
It also assumes that the thickness of the saltwater–freshwater interface is small relative to the thickness of the aquifer.

Experiments have shown that whenever the rise height, *z*, exceeds a critical value, the saltwater interface accelerates upward toward the well.

This critical rise height has been estimated to be in the range of 0.3*d* – 0.5*d*.

Taking the maximum allowable rise height to be 0.3*d* in the previous Equation leads to <u>the following Equation:</u>

$$Q_{max} = 0.6\,\pi\, d^2\, K_x\, \varepsilon \qquad (23)$$

EXAMPLE 6

A well pumps at 5 L/s in a 30-m thick coastal aquifer that has a hydraulic conductivity of 100 m/d.
How close can the saltwater wedge approach the well before the quality of the pumped water is affected?

Solution

<u>From the given data:</u>
Q_w = 5 L/s = 432 m^3/d, K_x = 100 m/d, and it can be assumed that ε = 0.025.

<u>The minimum allowable distance of the saltwater wedge from the well:</u>

$$d = \sqrt{\frac{Q_{max}}{0.6\,\pi\,\varepsilon\, K_x}} = \sqrt{\frac{432}{0.6\,\pi * 0.025 * 100}} = 9.6\ m$$

<u>Therefore,</u> the quality of pumped water will be impacted when the saltwater interface is less than or equal to 9.6 m below the pumping well.

SUMMARY

The groundwater flow equation	Numerical models	solve the three-dimensional flow equation
	Analytic models	solve the two-dimensional (Dupuit) approximations to the flow equation

Unconfined Flow between Two Reservoirs (Dupuit approximation)	
$\frac{K_1}{2} h^2 + (K_2 - K_1)\, b_2\, h = C_1 x + C_2$	$C_1 = \frac{K_1}{2\,L}(h_R^2 - h_L^2) + (K_2 - K_1)\frac{b_2}{L}(h_R - h_L)$
	$C_2 = \frac{K_1}{2}(h_L^2) + (K_2 - K_1)\, b_2\, h_L$
when (h) is calculated, then Q is: ➡ $Q = -C_1$	

Isotropic and Homogeneous		
Wells in a Confined Aquifer	**Wells in an Unconfined Aquifer**	
$T = \frac{Q_w}{2\,\pi\,(S_1 - S_2)} \ln\frac{r_2}{r_1}$	$T = \frac{Q_w}{2\,\pi\,(S'_1 - S'_2)} \ln\frac{r_2}{r_1}$	
	$S'_1 = S_1 - \frac{S_1^2}{2\,H}$	$S'_2 = S_2 - \frac{S_2^2}{2\,H}$
T = K * b	T = K * H	
The equation of an unconfined aquifer is the same result that was derived for a confined aquifer except that the modified drawdowns are used.		
The cone of depression is the drawdown surface surrounding the well.	In the common case where the drawdowns are small compared with the aquifer saturated thickness, ($S_1^2 \ll 2\,H$) and ($S_2^2 \ll 2\,H$); then, the modified drawdowns, S'_1 and S'_2, can be replaced by the actual drawdowns, S_1 and S_2.	
The piezometric head, φ, is independent of θ (radially symmetric) and independent of time.		
There is no recharge of the confined aquifer.		

Saltwater is denser than freshwater.

The zone of salinity transition (the transition zone) is the blurred interface between the freshwater and saltwater zones.

The saltwater wedge is the volume of water below the saltwater interface.

The Ghyben–Herzberg approximation is a method to determine the location of the saltwater interface (the pressure distribution is assumed to be hydrostatic within any vertical section of the aquifer, which assumes that the streamlines are horizontal).

$z = \frac{h}{\epsilon}$	$\epsilon = \frac{\rho_s - \rho_f}{\rho_f}$	$z \approx 40\ h$ (z = 33 : 50)
Ghyben–Herzberg equation	The buoyancy factor	The saltwater interface is at a distance below sea level = 40 times the elevation of the water table above sea level.

Employing the Dupuit approximation (horizontal flow and the horizontal piezometric head gradient = the slope of the water table):	For the water-table profile:	The finite width, W, of freshwater flow at the coastline (Glover Solution):
$Q = \frac{K}{2\ \varepsilon\ L}\ h_L^2$	$h = h_L \sqrt{\frac{x}{L}}$	$Q = \frac{K}{2\ \varepsilon\ (L - W)}\ h_L^2$

The depth of the saltwater interface below sea level, z, at any distance, x, from the coastline:		
$z = \sqrt{\frac{2}{\varepsilon\ K}\ (Q\ x - C)}$	→	$C = 0$ corresponds to applying the Ghyben–Herzberg approximation at the coastline.
	→	$C = -QW$ corresponds to applying the more realistic Glover solution at the coastline.

Upconing is the process by which the saltwater interface rises in response to pumping.

$$z = \frac{Q_w}{2\,\pi\,d\,K_x\,\varepsilon}$$

z: the rise height of the saltwater interface in response to pumping.

Whenever the rise height (z) exceeds a critical value, the saltwater interface accelerates upward toward the well.

z = 0.3d – 0.5d

Taking the maximum allowable rise height to be 0.3d:

$$Q_{max} = 0.6\,\pi\,d^2\,K_x\,\varepsilon$$